DEV BHoomi Institute of Technology

Chakrata Road, Navgoun
Manduwala, Uttarakhand

Programs: B.Tech. (Civil Engineering)

Structural Analysis Lab
Laboratory Manual

Prepared by
Lakshman Singh,
Asst.Professor,
Civil Engineering Department
LIST OF EXPERIMENTS

1) Experiment on a 2 hinged arch for horizontal thrust and influence line for horizontal thrust.

2) Experimental and analytical study of a 3 bar pin jointed truss.

3) Experimental and analytical study of deflection and unsymmetrical bending of a cantilever beam.

4) Begg defometer- verification of Muller Breslau principle.

5) Experimental and analytical study of an elastically coupled beam.

6) Sway in portal frames- demonstration.

7) To study the cable geometry and statics for different loading condition.

8) To plot stress –strain curve for concrete. Use of mechanical and electrical strain and stress gauge.
EXPERIMENT NO - 1

AIM: Experiment on a 2 hinged arch for horizontal thrust and influence line for horizontal thrust.

Theory:

The horizontal thrust ‘H’ in the case of two hinged arch is given by:

\[
H = \frac{\int M_s y \, ds/E_1}{\int M_s y \, dx} = \frac{\int y^2 \, ds/E_1}{\int y^2 \, dx} \]

Where \( ds = dx \, \sec \theta \)

1 = 1_c \, \sec \theta \) and \( E_1/c \) is constant

Take the case of two hinged parabolic arch subjected to a concentrated load \( W \) at a distance of \( kl \) from A. (fig. 1)

In the parabolic arch, rise of arch ‘y’ at any section at a distance \( x \) from A is given by \( y = A \, x \, (1-x) \) (considering support as origin)

When \( x = \frac{1}{2}, \) \( y = h, \) putting these values We find \( A = 4h/1^2 \)

Therefore \( y = 4h/1^2 \, (1-x) \)

And \( V_A = W \, (1-k) \)

\( V_B = W \, k \)
\[ K_l = 1 \\
\int M_s y\, dx = \int w (1-k) x 4hx (1-x)dx/l^2 + \int W_k 4hx (1-x)/l^2 dx \]

\[ = \frac{1}{3} hWk^2 (l-k) (l+k-k^2) l \]

\[ \int y^2 dx = \int [4hx (1-x)/l^2]^2 \, dx = \frac{8}{15} h^2 l \]

\[ H = \int y^2 dx = \frac{8}{15} h^2 l \]

Put \( W = l \) and solve to get influence line ordinate for

\[ H = \frac{5}{8} kl (k^3-2k^2+l) \]

\[ H_{\text{max.}} = \frac{25}{128} \left( \frac{1}{x} \right) \]

**Apparatus:**

Two hinged arch model, weights, scale, dial gauge etc.

**Procedure:**

1. Place the 1kg load on the first hanger position, move the lever the lever into contact with a 100gm hanger on the ratio 4:1 position adjust the dia gauge to zero.
2. Add 10kg to the 1kg hanger without shock and observe the dial reading.
3. Restore the dial to zero reading by adding loads to the lever hanger, say the load is \( W \).
4. The experimental value of the influence ordinate at the first hanger position on is than \( 4W/10 \).
(5) Repeat the process for all other loading position and tabulate and plot the influence ordinates.
(6) Compare the experimental values with those given by above formula.

**Precautions:-**

(a) Apply the load without jerk.
(b) Perform the expt. Away from vibration.

**Observations and calculations:-**

<table>
<thead>
<tr>
<th>Span of the arch, l =</th>
<th>mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>General rise of arch, h =</td>
<td>mm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load (kg)</td>
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<tr>
<td>Influence coordinate (4W/10)</td>
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<tr>
<td>Calculated ordinate</td>
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</tr>
</tbody>
</table>

**Results:-**

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Balancing Weight</th>
<th>Hanger Position</th>
<th>Horizontal Thrust Experiment</th>
<th>K= position of wt app</th>
<th>Span of Arch</th>
<th>Calculated Thrust</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wa/10b</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

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**Precautions:-**

- (c) Apply the load without jerk.
- (d) Perform the expt. Away from vibration.

**Observations and calculations:-**

Span of the arch, \( l = \) mm

General rise of arch, \( h = \) mm

<table>
<thead>
<tr>
<th>Load position</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load (kg)</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Influence coordinate ((4W/10))</td>
<td></td>
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<td></td>
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<tr>
<td>Calculated ordinate</td>
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<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Results:-**

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Balancing Weight</th>
<th>Hanger Position</th>
<th>Horizontal Thrust Experiment</th>
<th>K= position of wt app</th>
<th>Span of Arch</th>
<th>Calculated Thrust</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Wa/10b</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Diagram:-

FIG. 1
EXPERIMENT NO - 2

Object:-

Experimental and analytical study of a 3 bar pin jointed truss.

Theory:-

Following notation will be used:-

$L_1$ = length of member AD
$L_2$ = length of member CD
$L_3$ = length of member BD
$a$ = distance AB
$b$ = distance BC
$w$ = applied load W at D
$V$ = vertical displacement of D under an applied load W.
$U$ = horizontal displacement of D under an applied load W.
$T_1$ = tensile force in member AD
$T_2$ = Tensile force in member CD
$T_3$ = Tensile force in member BD

The values of $U$, $V$, $T_1$, $T_2$ and $T_3$ are given by the following expressions:-
W \quad (N_1a - N_2b)

U = \quad \frac{W}{(N_1a^2 - N_2b^2)}

V = \quad \frac{x}{A_1 E_1}

T_1 = (L_3 \cdot V - a \cdot U) \frac{A_1 E_1}{(L_1)^2} \quad (i)

T_2 = (L_3 \cdot V + b \cdot U) \frac{A_2 E_2}{(L_2)^2}

T_3 = (L_3 \cdot V \cdot A_3 E_3) \frac{1}{(L_3)^3}

where \quad N_1 = \quad x \quad \frac{1}{A_1 E_1}

where \quad N_2 = \quad \frac{A_3 E_3}{1}

where \quad N_3 = \quad \frac{x}{\frac{1}{A_3 E_3}}

where \quad \frac{AE}{L} is the stiffness of the member and is defined as force per unit deformation (extension). The stiffness (K), can be determined for each spring by taking out the spring and suspending from is loads of 1kg, 3kg, 4kg and 5kg (10, 30, 40 and 50N) in succession and measuring the extension in each case. A graph plotted between load and extension, which shall be a straight line with in elastic limit and the values of K which is the slope of the line can be found from this graph.
After knowing the values of U, V from equation (i) and (ii), values of $T_1$, $T_2$ and $T_3$ can be obtained from equations (iii), (iv) and (v) and shall be compared with the measured values in spring balance of each member.

**Apparatus:-**

Three bar suspension system, deflection dial gauges, scale, weights etc.

**Procedure:-**

(a) Find out the stiffness (K) of each spring by isolation from the apparatus.
(b) Measure the distance a, b, L1, L2 and L3 with the help of a meter scale.
(c) Fix the dial gauges at point D to measure the vertical and horizontal displacements simultaneously for each addition of vertical load.
(d) Remove the slackness of the members by putting a load of 0.5kg (5N) load at joint D and treat the deflected position as the datum (Initialization).
(e) Note the deflections U and V and forces $T_1$, $T_2$ and $T_3$ at least for 3 sets of loads.

**Precautions:-**

(a) Spring stiffnesses should be found out very accurately.
(b) Distances a, b and length L1, L2 and L3 should also measured accurately.
(c) Tap the dial gauges before taking for vertical and horizontal displacements.

**Observations and calculations:-**

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Load (kg)(N)</th>
<th>Extension in spring (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>AD</td>
</tr>
<tr>
<td>1.</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>4.0</td>
<td></td>
</tr>
</tbody>
</table>

Prepared By: Mr. Lakshman Singh
**Table 17.2 Values of U, V, T₁, T₂, T₃**

<table>
<thead>
<tr>
<th>Value of</th>
<th>Load system 1</th>
<th>Load system 2</th>
<th>Load system 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₁</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₂</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T₃</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Result:-**

Compare the experimental and analytical values of T₁, T₂ and T₃ and give your comments.

**Diagram:-**

![Diagram showing a truss structure with labels T₁, T₂, T₃, L₁, L₂, L₃, and U.](image)
EXPERIMENT - 3

Object:-

Experimental and analytical study of defections for unsymmetrical bending of a cantilever beam.

Theory:-

In structural members subjected to flexure, the Euler Bernoulli’s equation \( \sigma/y = M/I = E/R \) is valid only if the applied bending moment acts about one or the other principal axis of the cross section. However a member may be subject to a bending moment which acts on a plane inclined to the principal axis (say). This type of bending does not occur in a plane of symmetry of the cross section, it is called unsymmetrical bending. Since the problem related to flexure in general differs from symmetrical bending, it may be termed as skew bending.

Every cross-section of a symmetric section has two mutually perpendicular principal axes of inertia, about one of which the moment of inertia is the maximum and about the other a minimum. It can be shown that the symmetric axis of cross-section is one of the principal axes and one at right angles to the same will be the other principal axis.

From the principal of mechanics, any couple which may cause bending moment at a section of a beam may be resolved in to two components. The component of the bending moment acting around x is \( M \cos \alpha \), while the one acting around the y-axis is \( M \sin \alpha \). The sense of each component follows from the sense of the total moment M. These moments may be used separately in the usual flexure formula and the compound normal stresses follow by superposition as follows:

\[ \sigma = \pm M_{xx}\cdot y/I_{xx} \pm M_{yy}\cdot x/I_{yy} \] .......................... (i)

One or the other of the principal axes of the cross-section.

(a) Bending Moment in a plane that is not coincident with either of the principal axes.
(b) Components of the bending moment in the plane of the principal axes.

For beams having unsymmetrical cross-section such as an angle (L) or a channel (I)
section, if the plane of loading is not coincident with or parallel to one of the principal axes, the bending is not simple. In that case it is said to be unsymmetrical or non-uniplanar bending.

In the present experiment for a cantilever beam of an angle section, the plane of loading is always kept vertical and the angle iron cantilever beam itself is rotated through angles in steps of 45°.

Consider the position of the angle section as shown in Fig. (2). The plane of loading makes an angle \( \phi \) with V-V axis of the section, which is one of the principal axes of the section. The components of the vertical load \( P \) along V-V and U-U axes are \( P \cos \phi \) and \( P \sin \phi \) respectively.

The deflections \( U \) and \( V \) along U-V and V-V axes respectively are given by

\[
y = P \sin \phi \cdot \frac{L^3}{3EI_{VV}} \quad \text{(ii)}
\]

\[
y = P \cos \phi \cdot \frac{L^3}{3EI_{UU}} \quad \text{(iii)}
\]

And the magnitude of resultant deflection \( 00' \), is given by

\[
\sqrt{\Delta^2 y + \Delta^2 \zeta} \quad \text{(iv)}
\]

And its direction is given by

\[
\beta = \tan^{-1} \frac{\zeta}{y} \quad \text{(v)}
\]

Where \( \beta \) is the inclination of the resultant deflection with the U-U axes. This resultant displacement is perpendicular to the neutral axis n-n (Fig. 3) but not in the plane of the load \( P \).

In Fig. the following notation has been used:-

\[
00' = D
\]

\[
0^*p = DV
\]

\[
0p = DU
\]

\[
0^*Q = DX
\]

\[
0Q = DY
\]
\[ \tan \beta = \frac{\zeta}{y} = \frac{0'P/0P}{y} = \frac{P \cos \phi \cdot L^3}{3E l_{uu}} \]

\[ \tan \beta = \tan \beta = \frac{\zeta}{y} = 0'P/0P = \frac{P \sin \phi \cdot L^3}{3E l_{vv}} \]

\[ \tan \beta = \frac{l_{vv}}{l_{uu}} \cot \phi \]

Therefore the magnitude of angle \( \beta \) can be calculated from equation (vi).

For the angle section used in the present experiment \( l_{uu} \) and \( l_{vv} \) can be known from the tables of Bureau of Indian Standard hand book for properties of standard section. Therefore a given angle \( \phi \), the magnitude of angle \( \beta \) can be calculated from equation (vi).

The horizontal and vertical components of the deflection can be calculated on the basis of the geometry available as shown in Fig. 4. It can be seen.

\[ \zeta = P \cos (\phi + \beta) \]

\[ > \]

\[ \psi = P \sin (\phi + \beta) \]

Similarly

\[ \zeta = P \cos (\phi + \beta) \]

\[ = P \cos \phi \cdot \sin \beta \]

\[ \psi = P \sin \phi \cdot \cos \beta \]

\[ = P \sin \phi \cdot \cos \beta \]

Therefore the procedure of calculating the deflections would be:-

(a) Calculate \( U \) and \( V \) using equations (ii and iii)
Compute using equations (iv)
(c) Compute $\beta$ using equation (vi)
(d) Calculate the required values of $x$ and $y$ using equations (viii) and (ix).

Apparatus:-

Cantilever beam having an equal angle section. The beam is fixed at one end with possibility of rotation of 45° intervals and clamped. At the free end, the loading arrangements are such that vertical loading is always ensured, dial gauges, weights etc.

Procedure:-

(a) Clamp the beam at zero position and put a weight of 500gms (5N) on the hanger and lake the zero loading on the beam to activate the member.

(b) Set the dial gauges to zero reading to measure vertical and horizontal displacements at the free end of the beam.

(c) Load the beam in steps of 1kg (10N) up to 8kg (80N) and note the vertical and horizontal deflections each time.

(d) Repeat the steps (a) to (c) turning the beam through 45° intervals. Problem of unsymmetrical bending will arise only in those cases where the legs of the angle section are in horizontal and vertical positions. In those cases both vertical and horizontal deflections need be measured.

(e) Compute the theoretical deflections and compare with those measured experimentally.

Precautions:-

(1) Take care to see that you do not exert force on the free end of the cantilever beam.
(2) Put the load on the hanger gradually without any jerk.
(3) Perform the test at a location which is free from vibrations.

Observations and calculations:-

(a) Material of beam – mild steel.
(b) Young's modulus of the material ($E$) = $2 \times 10^6$ kg/cm² ($2 \times 10^5$ N/mm²)
(c) Span of cantilever beam ($L$) = cm
(d) Sectional properties
i. Size = 
ii. $I_{xx} = \text{mm}^4$
iii. $I_{yy} = \text{mm}^4$
iv. $I_{uu} = \text{mm}^4$
v. $I_{vv} = \text{mm}$
vi. Area = $\text{mm}^2$

### Table (B-19.1) Deflections

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Angle (°)</th>
<th>Load (kg) (N)</th>
<th>Observed Deflections (mm)</th>
<th>Measured Deflections (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>1.</td>
<td>0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2.</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>90</td>
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<td>4.</td>
<td>135</td>
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<td>5.</td>
<td>180</td>
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<tr>
<td>6.</td>
<td>225</td>
<td></td>
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<tr>
<td>7.</td>
<td>270</td>
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<tr>
<td>8.</td>
<td>315</td>
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<td></td>
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</tr>
</tbody>
</table>

**Results:**
EXPERIMENT – 4

Aim: - To verify the Muller Breslau theorem by using Begg’s deformator set.

Apparatus: - Muller Breslau Principal Begg’s Deformeter, Micrometer, Microscope, Functioning plugs, Influence line ordinates, Drawing board, Pins etc.
Theory : Various methods are available to verify Muller Breslau’s Principle such as Brass wire Method, Begg’s Deformeter, Eney Deformeter and Gottschalk Continostat method. The Begg’s Deformeter Method is usually the most satisfactory experimental method. This principle states “the ordinates of the influence lines for any stress element (such as axial stress, shear moment or reaction) at any section of the structure are proportional to those of the deflection curve which is obtained by removing the restraint corresponding to that element from the structure and introducing in its place a deformation into the primary structure which remains. This principle is applicable to any type of structure whether statically determinate or indeterminate. Incase of indeterminate structures this principle is limited to structures, the material of which is elastic and follows Hook’s law.

The application of Begg’s deformer apparatus involves the use of the relation. 

\[ F_2 = k p_n \left( \frac{\delta n}{\delta_2} \right) \]

\( F_2 \) = desired force component at point 2, produced by \( p_n \).
\( p_n \) = force assumed to be acting on structures at point \( n \).
\( k \) = a constant; scale reduction factor if \( F_2 \) is a moment component or unity if \( F_2 \) is a thrust or shear component.
\( \delta n \) = deformation introduced at point 2 in the direction of \( F_2 \).
\( \delta_2 \) = deformation introduced at point \( n \) caused by \( F_2 \) and measured in the direction of \( p_n \).

Procedure:

i) Choose the problem for the study e.g. portal frame of equal / unequal legs, beams, trusses or any other structures.

ii) Choose the scale reduction factor for the linear dimensions that will give a model of
a size to be made easily and used conveniently. For most structures this factor will be of such a value as to allow the model to be cut from a standard sheet.

iii) Select the material for model e.g. plastic sheet/Acrylic/Perspex or any other desired sheet. The width and thickness should remain same throughout the length used.

iv) Cut the model in the selected shape /size . Mark the center line throughout the model length otherwise every time one should have to find out the centre.

**Observation Table:**

<table>
<thead>
<tr>
<th>S. No</th>
<th>Points</th>
<th>Thrust</th>
<th>Shear</th>
<th>Moment</th>
<th>Calibration constant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table - 2**

Influence Line Ordinates:

<table>
<thead>
<tr>
<th>S. No</th>
<th>Points</th>
<th>Thrust</th>
<th>Shear</th>
<th>Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Calculated</td>
<td>Observed</td>
<td>Calculated</td>
</tr>
<tr>
<td>1. 2. 3.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Calculation:** - The ordinate of influence line diagram obtained experimentally:

The ordinate of influence line diagram obtained analytically:

**Result:** - The Muller Breslau Principle is verified experimentally and analytically.

**Precaution:** - i) Apply the loads without any jerk.
ii) Measure the deflection to the nearest of a millimeter.

iii) Perform the experiment at a location, which is away from any

iv) Avoid external disturbance.

v) Ensure that the supports are rigid.
EXPERIMENT – 5

Object:

Experimental and analytical study of elastically coupled beam.

Theory:

Let the internal forces in the three rods Ad, BE, and CF be $R_1$, $R_2$ and $R_3$ respectively. Lengths $AD = BE = CF = L$

Lengths $AB = BC = DE = EF = L$

$I = \text{Moment of inertia of the cross-section of the beam ABC and the beam DEF.}$

$E = \text{Young's Modulus of Elasticity of the materials of the beam ABC and beam DEF. } E_1, E_2 \text{ and } E_3 = \text{Modulus of elasticity of the material of rods AD, BE and CF respectively. } A_1, A_2 \text{ and } A_3 = \text{Cross-sectional area of rods AD, BE and CF respectively.}$

$$DD_1 = y_1$$

$$EE_1 = y_2$$

$$FF_1 = y_3$$

$$EE_2 = (y_1+y_3)/2$$

$$E_2E_1 = (11/96) (WL^3/EI) – (R_2L^3/6EI)$$

Case I

When support B exists, beam ABC becomes in-operative. The central deflection at point
E, of beam DEF, due to load – at G and force R₂ at E, Relative to the deflected portions of points D and F given by:

\[ E = (11/96) (W L^3/E l) - (R_2 L^3/6E l) \] ........................ (i)

Let \( y_1 \) and \( y_3 \) be the elongations of the rods AD and CF respectively. The total deflection of points E relative to its original positions is

\[ (y_1 + y_3)/2 + (11/96) (W L^3/E l) - (R_2 L^3/6E l) \]

and this should be equal to the elongation of the rod BE i.e. equal to \( y_2 \).

Now \( y_1 = R_1 L/A_1 E_1 \), \( y_2 = R_2 L/A_2 E_2 \) and \( y_3 = R_3 L/A_3 E_3 \)

Substituting the values of \( y_1 \) and \( y_2 \) in eqn. (ii)

\[ R_2 L/A_2 E_2 = 1/2 \{(R_1 L/A_1 E_1) + (R_3 L/A_3 E_3)\} + (11/96) (W L^3/E l) - (R_2 L^3/6E l) \] ........................ (iii)

Also

\[ R_1 + R_2 + R_3 = W \] .......................... (iv)

Taking moments of all forces about F

\[ R_1 L - W \{3/2L\} + R_2 L = 0 \]

Or \[ R_1 = 3/4W - R_2/2 \] .......................... (v)

Solving eqn. (iii), (iv) and (v) simultaneously for \( R_1 \), \( R_2 \) and \( R_3 \) we obtain

\[ R_2 = \frac{3K_1 + K_3 +(11/12)L^3/E l}{x} \]

Where \( K_1 = L/A_1 E_1 \), \( K_2 = L A_2 E_2 \) and \( K_3 = L A_3 E_3 \)

Case 2

If the support B were not there, beam AC will also deflect due to the load R₂ applied at its centre. Here the total central deflection of point E, relative to its original positions, as given by the expression (ii) is equal to the elongation of member BE plus the central deflection of beam ABC.
\[ R_2 \frac{L}{A_2 E_2} + R_2 \frac{L^3}{6E_1} = \frac{1}{2} \{ R_1 \frac{L}{A_1 E_1} + R_3 \frac{L}{A_3 E_3} \} + \frac{(11/96)(W L^3)}{E_1} \]

\[-R_2 \frac{L^3}{6E_1} \]

\[ \text{........................... (iii A)} \]

Solving equs. (iii A), (iv) and (v) for \( R_1, R_2 \) and \( R_3 \) simultaneously.

\[ w = \frac{3K_1 + K_3 + (11/12) L^3}{E_1} \]

\[ x \]

\[ R_2 = \]

In expressions (vi and (vii) the quantity \( K = \frac{L}{A.E} \) for any spring (used as suspension rod here) is the extension of the spring per Kg. (N) of weight. This may be determined for individual springs.

**Apparatus:-** Elastically coupled beam model weights, scale etc.

**Procedure:-**

(a) Plot graphs between load applied and extension in each spring. From the graph determine the value of stiffness \( K \) (extension per unit load) for each spring.

(b) Tighten the screw at top for case 1 to make the supports rigid, load the beam DEF at quarter point and measure extensions of springs. Start with initial load of 1kg. (10N) increments of 1kg (10N) and maximum load of 4kg (40N).

**Precautions:-**

(a) Increase the load on the spring gradually while finding the value of \( K \) of individual spring.

(b) Load the lower beam without nay jerk.

(c) Measure the extensions of the springs very accurately.

**Observation and calculations:**

**Table. \( K = \frac{L}{A.E} \)**

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Spring AD</th>
<th>Spring BE</th>
<th>Spring CF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Prepared By: Mr. Lakshman Singh
Table 2. Observed Values of Reactions.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Reading at</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R₁ (Reaction)</td>
</tr>
<tr>
<td>Case-1.0 kg</td>
<td>2.10 N (1kg)</td>
</tr>
</tbody>
</table>

Table 3. Comparison of Results

<table>
<thead>
<tr>
<th>Applied Load</th>
<th>R₁</th>
<th>R₂</th>
<th>R₃</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-1</td>
<td></td>
<td></td>
<td></td>
<td>Observed Calculated</td>
</tr>
<tr>
<td>10 N (1kg)</td>
<td></td>
<td></td>
<td></td>
<td>Observed Calculated</td>
</tr>
<tr>
<td>20 N (1kg)</td>
<td></td>
<td></td>
<td></td>
<td>Observed Calculated</td>
</tr>
<tr>
<td>30 N (1kg)</td>
<td></td>
<td></td>
<td></td>
<td>Observed Calculated</td>
</tr>
</tbody>
</table>
EXPERIMENT – 6

Object:-

Sway in portal frames-demonstration.

Theory:-

Sway in portal frames may be due to the following reasons:-

i. Eccentric or unsymmetrical loading on the portal frame (Fig. i).
ii. Unsymmetric outline of portal frame (Fig. ii).
iii. Different end condition of the columns of the portal frames (Fig. iii)
iv. Non-Uniform section of the members of the frames (Fig. iv)
v. Horizontal loading of the supports of the frames (Fig. v)
vi. Settlement of the supports of the frames (Fig. vi)
vii. A combination of the above (Fig. vii)

In such cases, the joint translations become additional unknown quantities. Some additional conditions will, therefore, be required for analyzing the frames. The additional conditions of equilibrium are obtained from the consideration of the shear force exerted on the structure by the external loading. The horizontal shear exerted by a member is equal to the algebraic sum of the moments at the ends divided by the length of the members. Thus the horizontal shear resistance of all such members can be found and the algebraic sum of all such forces must balance the external horizontal loading. If any, see Fig. (viii)

Taking moments about top joint, expressions for horizontal shear at supports A and B will be

\[ M_{AB} + M_{BA} - P_a \]

\[ H_A = \frac{M_{DC} + M_{CD}}{L_{AB}} \]
After knowing $H_A$ and $H_D$, we can write horizontal equilibrium equation for the frame as $H_A + H_D + P = 0$

This gives additional equation required. It is called shear equation/shear condition. Hence $Q_A$ and $Q_B$ and (Sway) can be found.

Similarly for case given below (sec. fig. ix)

The shear equation can be written as $H_A + H_D + P = 0$

\[
M_{BA} + M_{AB} - \frac{P_a}{L}
\]

\[
H_A = \frac{M_{CD} + M_{DC}}{L}
\]

\[
H_P = \frac{M_{CD} + M_{DC}}{L}
\]

**Apparatus:-**

Portal frames model, weights, scale and gauges etc.

**Procedure:-**

i. First of all confirm about the support conditions.

ii. Note down the initial reading on the gauges and confirm that frame is horizontal prior to loading.

iii. Apply load care fully.

iv. Note down the horizontal deflection with the help of gauges.

v. Calculate theoretically the sway by either slope deflection method or moment distribution method.

vi. Compare both values and find out error.
EXPERIMENT- 7

Object:-

To study the cable-geometry and static’s for different loading conditions.

Theory:-

Suspension bridges are used for highways, where the span of a bridge is more than 200m. Essentially, a suspension bridge (Fig. 1) consists of the following elements.

i. The cable,
ii. Suspenders
iii. Decking, including the stiffening girder.
iv. Supporting tower
v. Anchorage

The traffic load of the decking is transferred to main cable through the suspenders. Since the cable is the main load bearing member, the curvature of the cable of an unstiffened bridge change as the load moves on the decking. To avoid this, the decking is stiffened by provision of either a three hinged stiffened girder. The stiffened girder transfers a uniformly distributed or equal load to each suspender, irrespective of the load position on the decking. The suspension cable is supported on either side. There are two arrangements generally used. The suspension cable may either pass over a smooth frictionless pulley and anchored to the other side, or it may be attached to a saddle placed on rollers. In the former case the tension in the cable on the two sides of the pulley are equal while in the latter case, the horizontal components of the tension on the two sides are equal since the cable cannot have a movement relative to the saddle. The cable consists of the either wire, rope, parallel wires jointed with clips or eye bar links. The cable can carry direct tension only, and the bending moment at any point on the cable is zero. The suspenders consist of round rods or ropes with turn buckler so that adjustment in their lengths may be done if required. The anchorage consists of huge mass of concrete, designed to resist the tension of the cable. Let us take the case of a cable geometry having the cable chord horizontal. (fig. ii)
Due to symmetry, $V_A = V_B = WL/2$

Where $W$ is the intensity of loading $L$ is the span.

$C$ is the lowest point

Consider the equilibrium of the portion $LA$ to left of $C$ (Fig. iii). The left portion is in equilibrium under three forces. (i) the cable tension $T$ at $A$ (i.e) the resultant of $H$ and $V_A$. (ii) The external load $WL/2$ acting at $L/4$ from $C$, and (iii) The horizontal cable tension $H$ at $C$. All these forces must meet at a point becomes a triangle $AA_1E$ becomes a triangle of forces which.

$$H/A_1E = T/AE = (WL/2)(1/AA_1)$$

$$H = WL/2.A_1E/AA_1 = WL/2.L/4.L/d = WL²/8d$$

Hence $H = WL²/8d$

For cable tension: -

$$T_A = \sqrt{V_A^2 + H^2}$$

$$T_B = \sqrt{V_B^2 + H^2}$$

As the cable chord is horizontal $V_A = V_B$ and hence

$$T_A = T_B = T = \sqrt{(WL/2)^2 + (WL²/8d)^2}$$

$$T = H \sqrt{l + 16d²/L²}$$

The inclination $\beta$ of $T$ with the vertical is given by ten $\beta = H/V = WL²/8d.2/WL = L/4d$

It should be remembered that the horizontal component of cable at any point will be equal to $H$.

**Shape of the cable:** -

To determine the shape of the cable under uniformly distributed load. Substituting the value of $H$. 

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(WL²8d) \( y = WLx/2 - Wx²/2 \)
\( y = 4dx/L² (L-x) \)

This is thus the equation of the curve with respect to the cable chord. The cable, thus takes the form of a parabola when subjected to uniformly distributed load of the parabola can be written, with C as origin as follows

\[ y = Kx² \]
\[ x = L/2 \text{ and } y = d \]
\[ K = y/x² = d/(L/2)² = \]
\[ 4d/L² \]
\[ y = 4d/L²x² \]

Consider an element of length \( ds \) of the curve, having co-ordinates \( x \) and \( y \). The total length ‘\( s \)’ of the curve is given by

\[ S = 2 \int_{0}^{L/2} \sqrt{1 + \left(\frac{du}{dx}\right)^2} \, dx \]

By solving this, we will get \( S = L + 8/3 \, d²/L \)

**Apparatus:**
- Cable geometry model, weights, gauges.

**Procedure:**

i. Apply the given load carefully
ii. Note down the induced pull in the cable
iii. Note down the forces in various suspenders from the spring balances provided.
iv. Measure the heights of various points of the cable above girder.
v. Plot the geometry of cable from the data taken from step (iv)
EXPERIMENT-8

Object: -

To Plot stress strain curve for concrete.

Theory: -

The results obtained from this experiment are used to study the behavior of concrete subjected to prolonged loading which has special importance as the concrete is not truly elastic material since it possesses the ability to ‘creep’ during and after the application of load. The phenomenon has been explained in fig. (i)

The modulus of elasticity of concrete and its corresponding compressive strength are required in the design calculations of concrete structures. In the field of Reinforced Cement Concrete design it is extensively used in the form of modular ratio.

The modulus of elasticity can be determined by measuring the compressive strain when a sample is subjected to a compressive stress. Indian Standards stipulate that height should be at least twice the diameter. Two extensometers should be used to check on eccentric loading and they should be mounted diametrical opposite.

The ultimate compressive strength of concrete shall be determined by testing three cubes at the time when the specimen is tested for determining the modulus of elasticity.

The modulus of elasticity is taken as the slope of the chord from the origin to some arbitrarily chosen point on the stress strain curve as shown in Fig. (ii). This is called secant modulus. Sometimes it is taken as slopes of the tangent at the origin or the slope of tangent at some arbitrary chosen stress (called tangent modulus) but the tangent at the origin is difficult to draw accurately.
Apparatus: -

Mixing pan, tamping bar, trowels, capping apparatus, Lamb’s extensometer with illuminated scale and telescope, a testing machine.

Procedure: -

(1) Take mix proportion as 1:2:4 by mass and water cement ratio as 0.50. Take 28kg of coarse aggregate, 14kg of fine aggregate and 7kg of cement and prepare the concrete mix.
(2) Prepare three 150mm cubes and three 150x300mm cylinders.
(3) Apply a proper capping to the top of the cylinders.
(4) Cure these specimens under water for 28 days.
(5) Test the three cubes in wet condition for compressive strength after 28 days of curing and determine the average compressive strength.
(6) **Mount the Lamb’s extensometer to a cylinder on its opposite sides and parallel to its axis.** The cylinders are also tested in wet condition.
(7) Load the cylinder at the rate of 14N/mm² per minute and at a regular interval of loading (generally two tones) record the extensometer readings.
(8) Calculate the stress and strains for each cylinder and plot stress-strain curves of concrete.
(9) Determine the modulus of elasticity (secant modulus) at 30 percent to cube strength.

Precautions: -

(1) Compacting strokes should be uniformly applied over the whole surface.
(2) The distance between the telescope and the scale should be so adjusted that a well defined image of the scale is obtained.
(3) The specimen for determining the modulus of elasticity should be loaded and unloaded three times to the stress of one third of ultimate strength.
(4) The compressive strain shall be read at intervals during second and third loading and if they differ by more than 5 percent. The loading should be repeated until strains of successive cycles do fall within limit.
(5) Two extensometers should be used to check on eccentric loading.